

Calculating Cosine and Sine

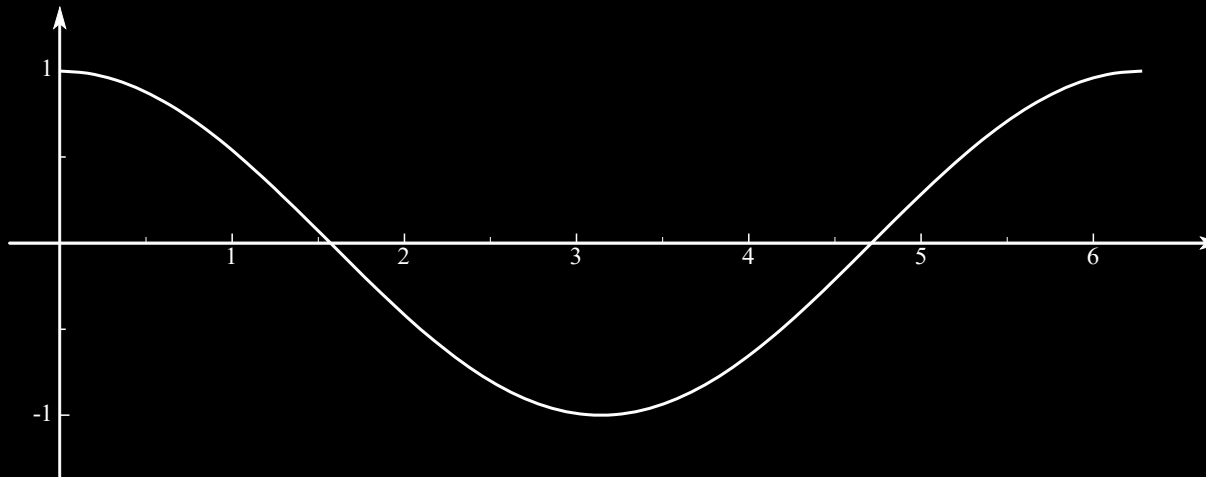
The Hackett Series

Cosine Example of Hackett Series (7 Iterations)

$$g(x) = 2x^2 - 1$$

$$f(x) = g(g(g(g(g(g(g(x \bmod \tau / 2^7)))))))$$

Output Graph:

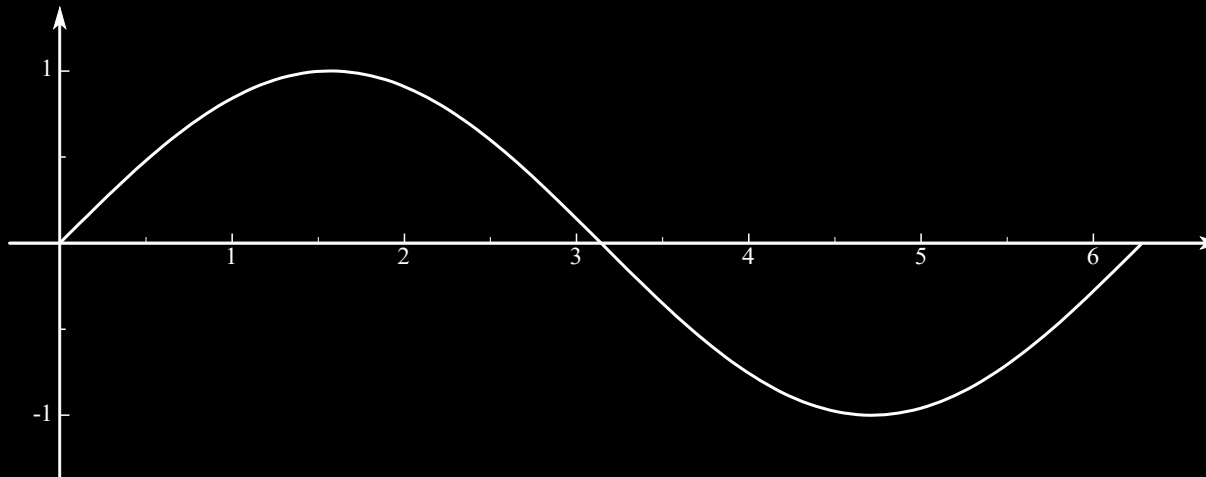


Sine Example of Hackett Series (7 Iterations)

$$g(x) = 3x - 4x^3$$

$$f(x) = g(g(g(g(g(g(g(x \bmod \tau / 3^7)))))))$$

Output Graph:



Hackett Series

$$\cos(x) = \lim_{n \rightarrow \infty} x_n, x_1 = \frac{x_1 \bmod \tau}{2^n}, x_n = 2x_{n-1}^2 - 1$$

$$\sin(x) = \lim_{n \rightarrow \infty} x_n, x_1 = \frac{x_1 \bmod \tau}{3^n}, x_n = 3x_{n-1} - 4x_{n-1}^3$$

$$\pi = \lim_{n \rightarrow \infty} 2^{n+1} \sqrt{2 - x_n}, x_1 = 0, x_n = \sqrt{2 + x_{n-1}}$$

$$\tau = \lim_{n \rightarrow \infty} 2^{n+2} \sqrt{2 - x_n}, x_1 = 0, x_n = \sqrt{2 + x_{n-1}}$$

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right), \cos(x) = \sin\left(x + \frac{\pi}{2}\right), \tan(x) = \frac{\sin(x)}{\cos(x)}$$

Cosine Source Code in C

```
int n = 10; // Precision
```

```
double cos(double x)
```

```
{
```

```
    x = fmod(x, tau) / (1 << n);
```

```
    for (int i = 0; i < n; ++i)
```

```
    {
```

```
        x = (x + x) * x - 1;
```

```
    }
```

```
    return x;
```

```
}
```

Sine Source Code in C

```
double sin(double x)
{
    x = fmod(x, tau) / pow(3, n);
    for (int i = 0; i < n; ++i)
    {
        x = 3 * x - 4 * x * x * x;
    }
    return x;
}
```

Pi Source Code in C

```
double pi()
{
    double p = 0;
    for (int i = 0; i < n; ++i)
    {
        p = sqrt(2 + p);
    }
    return sqrt(2 - p) * (1 << n + 1);
}
```

Tau Source Code in C

```
double tau()  
{  
    double p = 0;  
    for (int i = 0; i < n; ++i)  
    {  
        p = sqrt(2 + p);  
    }  
    return sqrt(2 - p) * (1 << n + 2);  
}
```